Lesson 7

Thermomechanical Measurements for Energy Systems (MENR)

Measurements for Mechanical Systems and Production (MMER)

A.Y. 2015-16

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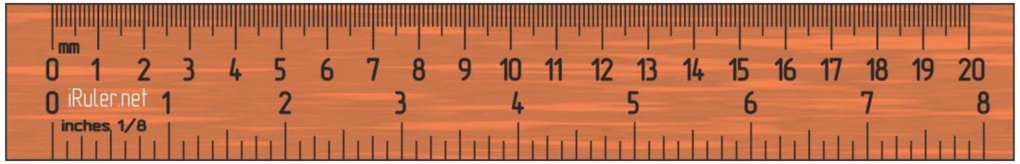
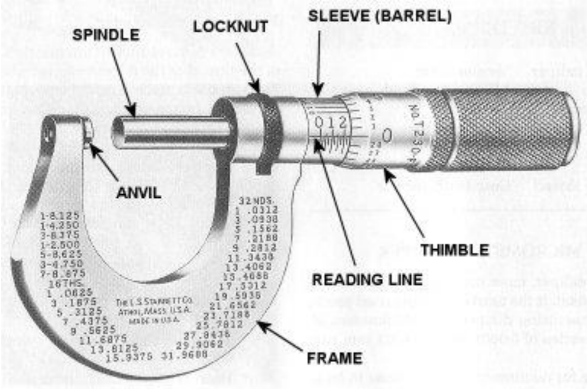
Length and displacement measurement

Many instruments are available to measure length and displacement in modern technology …

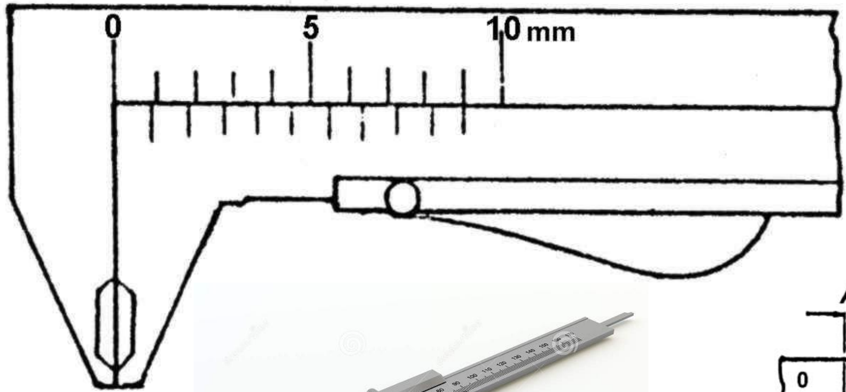
The most ancient is the ***graduated ruler:***

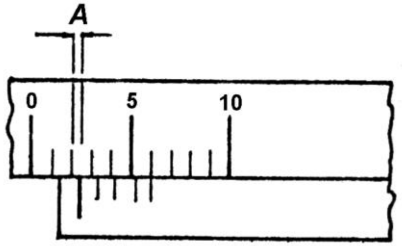


Rulers have an approximation of 1 mm, while ***Vernier Calipers*** have an approximation of 0,1 mm (over an average range of 200 mm) ! If you need more resolution, you have to use the ***Micrometer***, which has an approximation of 0,01 mm (over a range of 25 mm) !



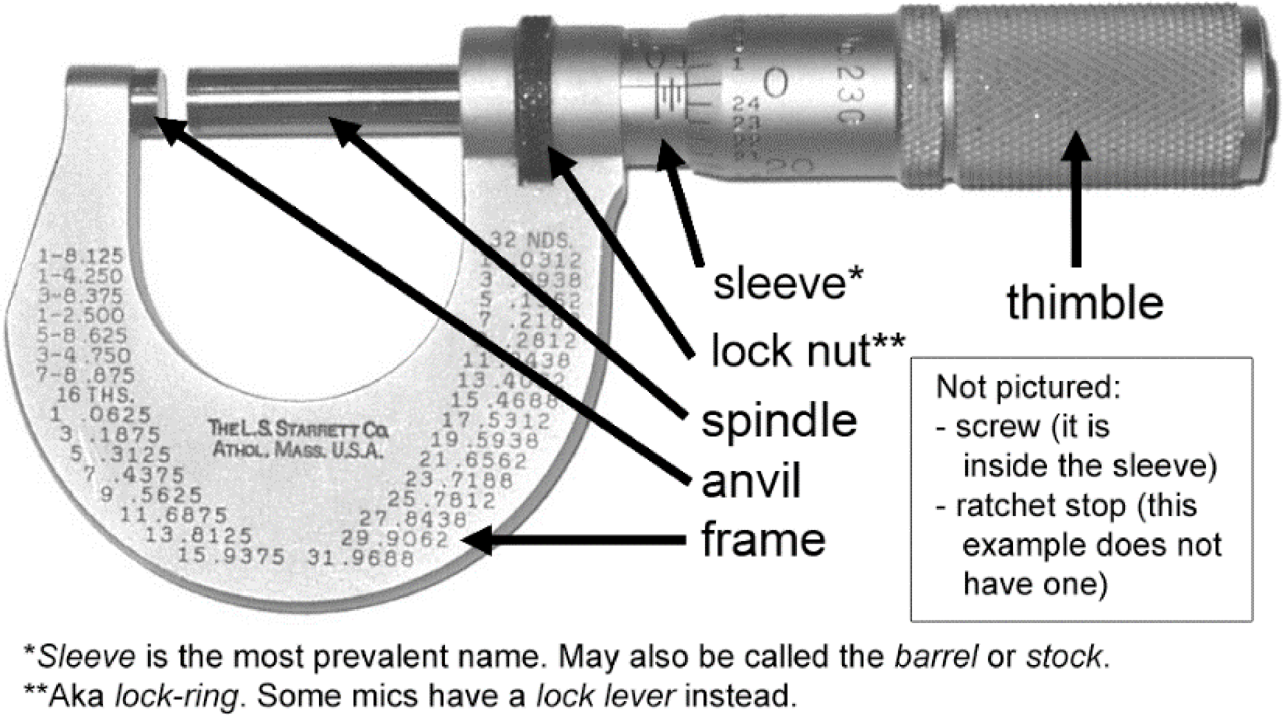
For small length the ***Vernier Caliper*** is used in the Lab:



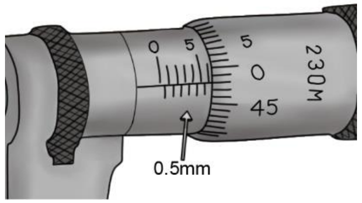


It subdivides 9 mm in 10 parts, each of which is therefore 0,9 mm long. When measuring, you count the first *mobile notch* coinciding with any *fixed notch* and you add that “tenth of millimeter” (A) to your reading !

The ***Micrometer (Palmer)*** is based on a "micrometric screw" with a *pitch* of 0.5 mm (*sleeve*)



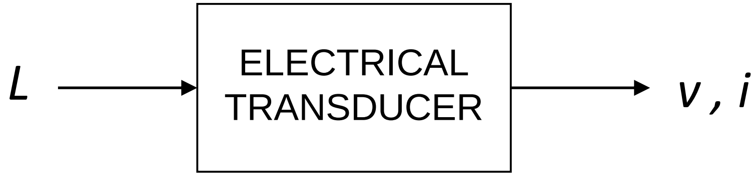
There are 50 notches on the rotating screw (*thimble*), therefore for every notch the screw advances:



$$\frac{1}{2}\cdot\frac{1}{50}=\frac{1}{100}\quad \text{mm},$$

which is also the approximation of the instrument !
Of course the range is limited by the instrument screw opening …

However in the industry, almost all the ***displacement transducers*** are <u>*electrical*</u>:



They are generally based on the <u>*Ohm's law*</u>: $V = Z \cdot I$

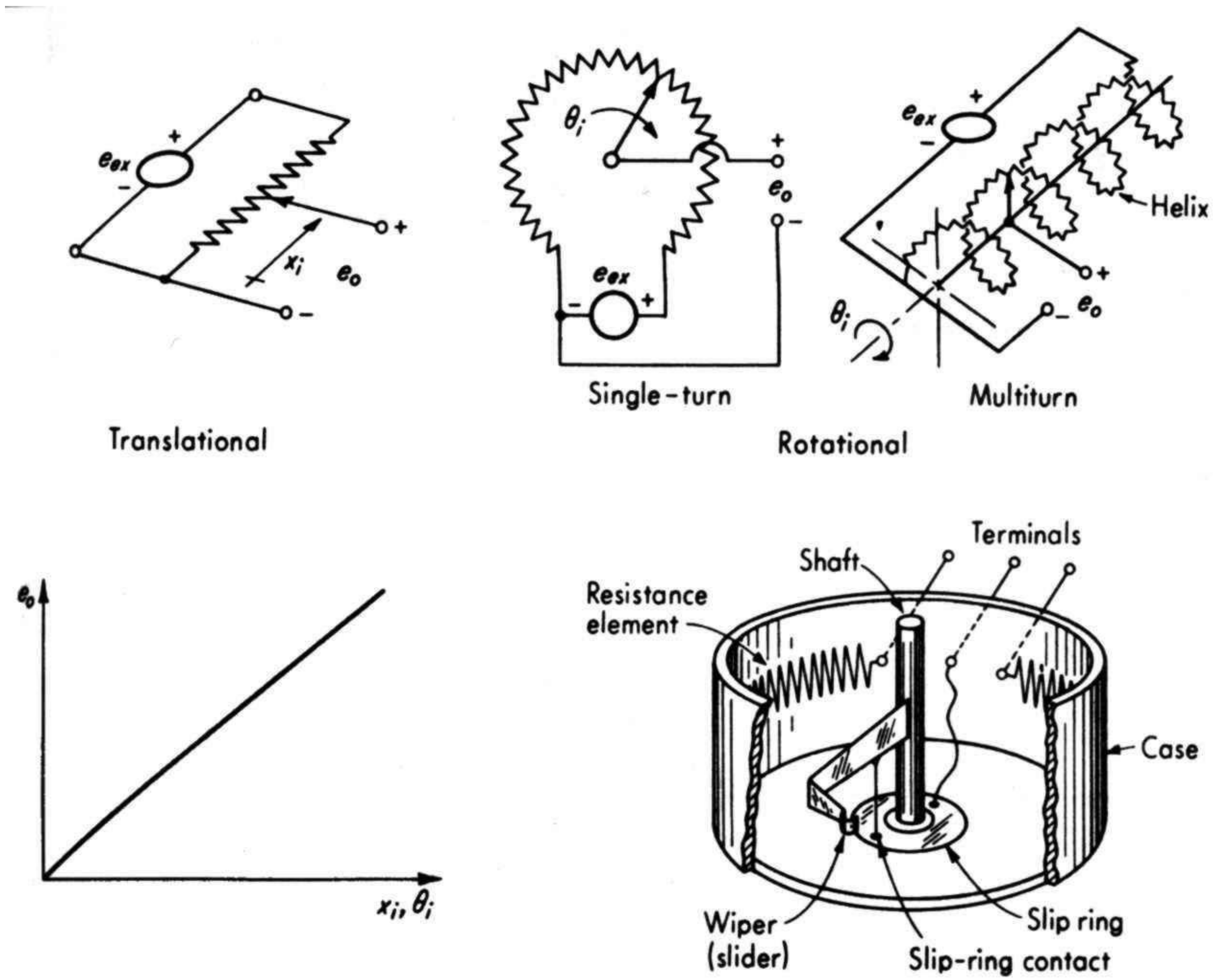
With the impedance: $Z=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$

We have already seen the ***potentiometers***, where $Z = R = \rho \cdot \frac{l}{S}$ and the graduation curve is: $$e_o = \frac{e_{ex}}{l} \cdot x$$

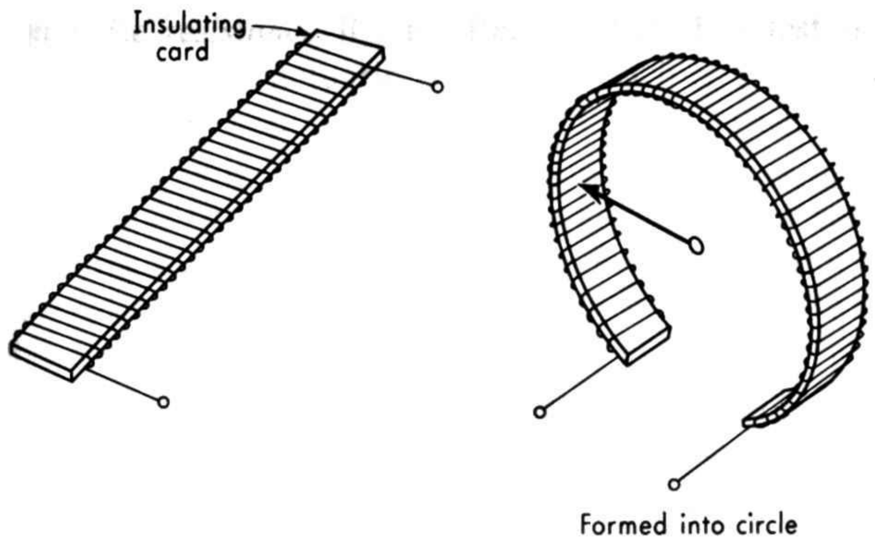
with a *constant sensitivity* expressed in [V/m] $S = \frac{de_o}{dx} = \frac{e_{ex}}{l}$

For ***rotational potentiometers*** we have:

$$e_o = \frac{e_{ex}}{2\pi} \cdot \vartheta$$

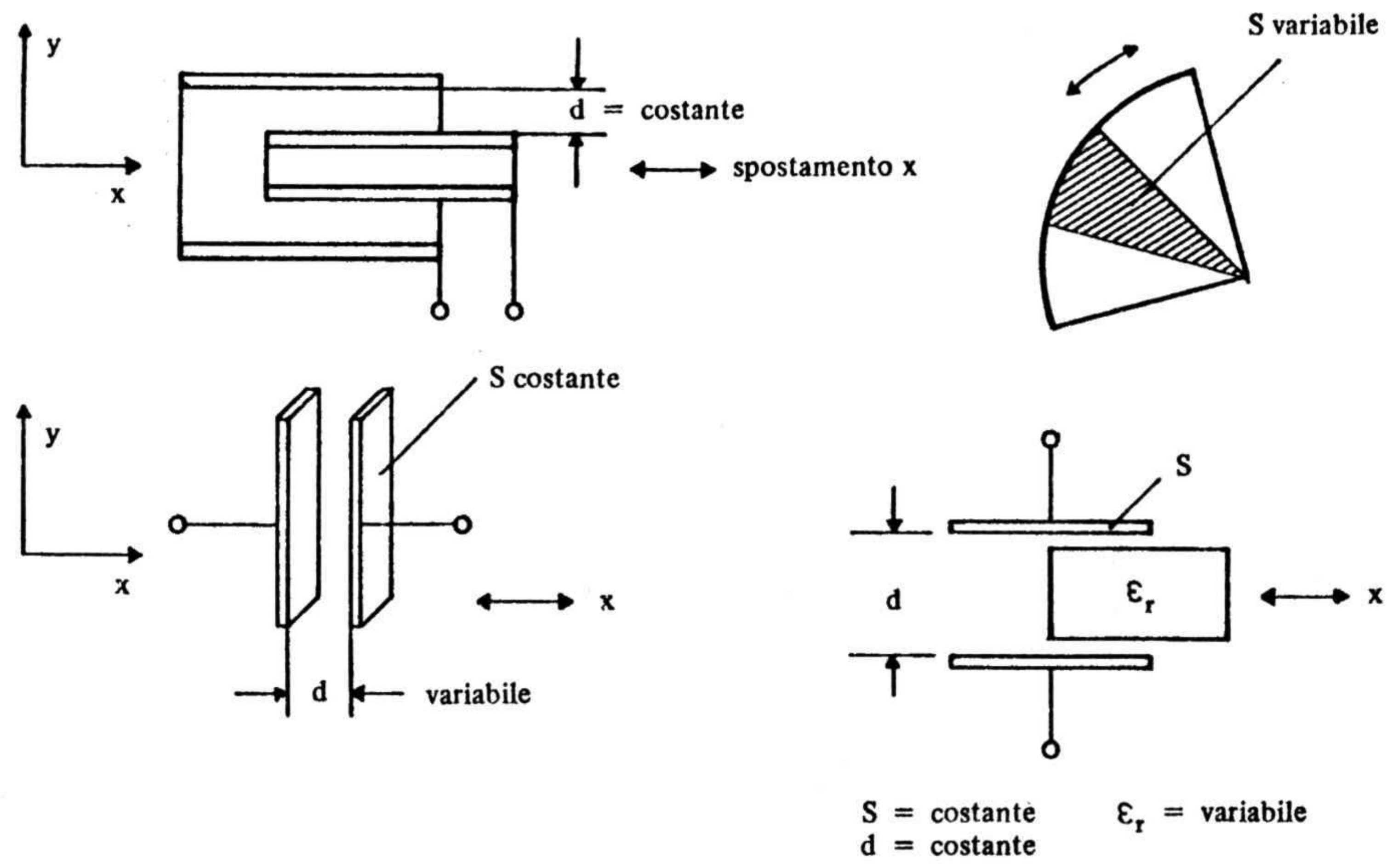


Potentiometer displacement transducer.



Construction of wirewound resistance elements.

Capacitive transducers are all based on the fundamental physical relationship: $C = \varepsilon_0 \varepsilon_r \frac{S}{d}$



We can vary the "distance *d*" but then we did <u>not</u> get a *constant sensitivity*:

$$\frac{dC}{d(d)} = -\varepsilon_0 \varepsilon_r \frac{S}{d^2}$$

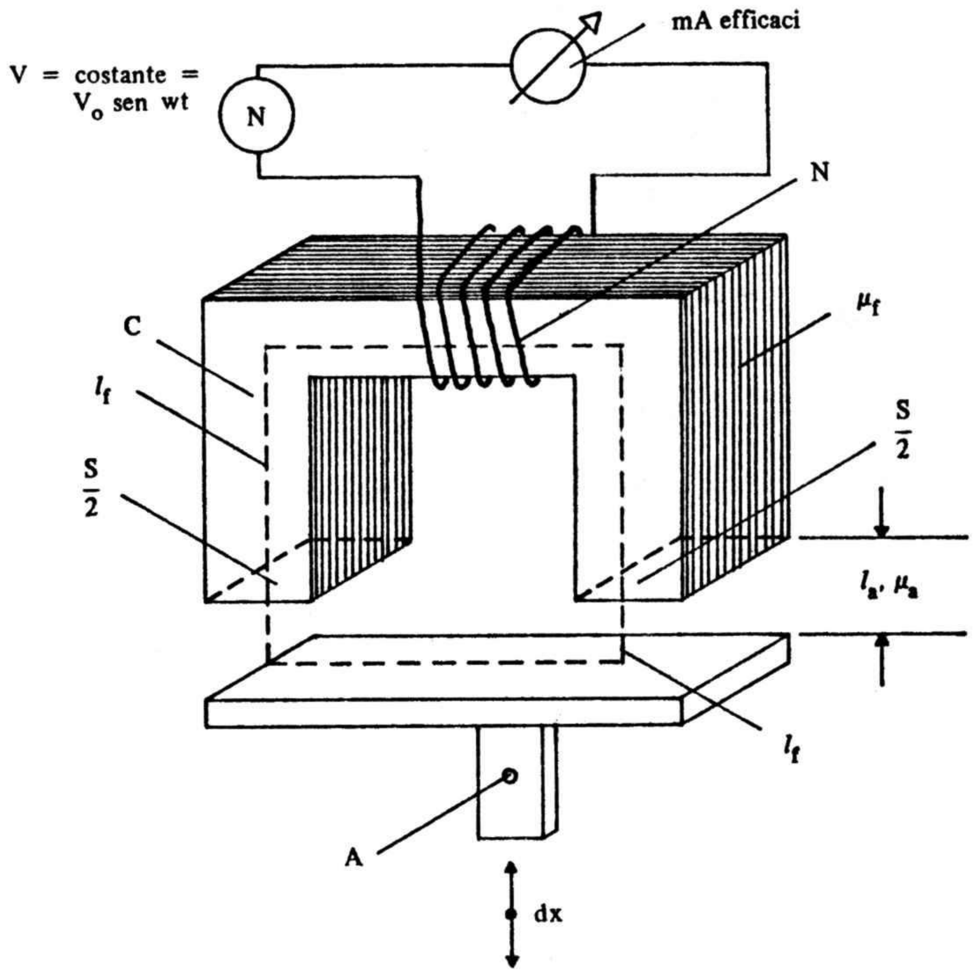
or we can vary the "plates surface *S*" (good for rotating capacitor transducers) and we get a *constant sensitivity*:

$$\frac{dC}{dS} = \varepsilon_0 \varepsilon_r \frac{1}{d}$$

or we can even vary the "dielectric constant ε_r" and we get a constant sensitivity as well:

$$\frac{dC}{d\varepsilon_r} = \varepsilon_0 \frac{S}{d}$$

Inductive transducers are a bit more complex, for example in the ***variable reluctance transducer*** the inductance



$$L = \mu_0 \mu_r \frac{N^2 S}{l}$$ is changing because of a variation of

the magnetic circuit reluctance $L = \frac{N^2}{\Re}$ where

$$\Re = \frac{l}{\mu_0 \mu_r S}$$ and it is: $\Re = \Re_f + \Re_a$

Because $\mu_f \geq 10000$ while $\mu_a \cong 1$ even if $l_f > l_a$

it basically results : $L \cong \frac{N^2}{\Re_a}$

If we supply the coil with a "c.a. voltage" : $V = V_0 sen\omega t$

we can apply $V = Z \times I$ where $\bar{Z} = R + j\omega L \cong j\omega L$

and the graduation curve is:

$$I_{eff} = \frac{2V_{eff}}{\omega N^2 \mu_0 \mu_a S} \cdot x$$

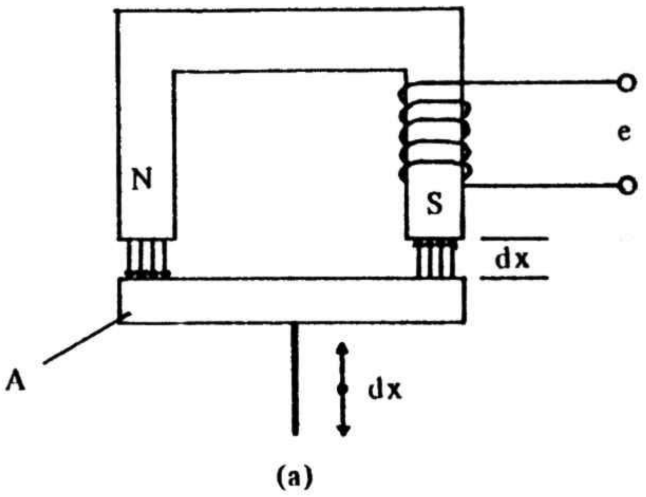
$$V = Z \cdot I \cong j\omega L \cdot I = j\omega \frac{N^2}{\Re_a} \cdot I = j\omega N^2 \mu_0 \mu_a \frac{S}{l_a} \cdot I$$

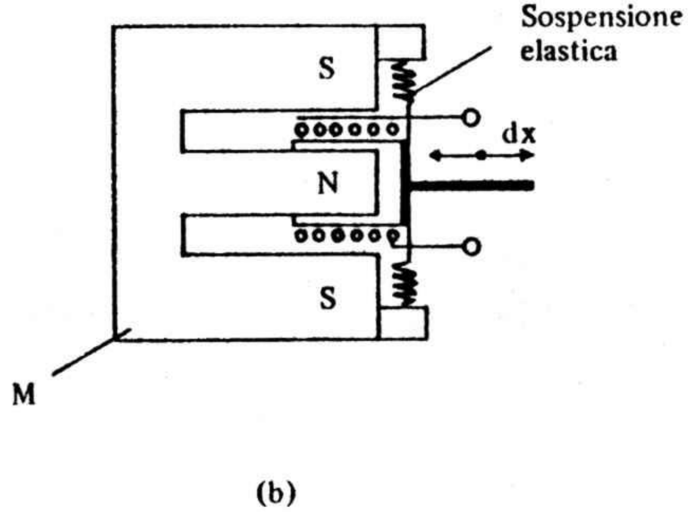
with a *constant sensitivity* :

$$S = \frac{\Delta I}{\Delta x} = \frac{2V_{eff}}{\omega N^2 \mu_0 \mu_a S}$$

There are also *simpler inductive transducers* which do not need a voltage supply, because they are based on *permanent magnets* and on the *Faraday's law* :

$$e_u = -N\frac{d\Phi}{dt} = -N\frac{d\Phi}{dx}\frac{dx}{dt} = -N\frac{d\Phi}{dx}\dot{x}$$





However, the output signal is proportional to the *velocity of the displacement* $\dot{x}$ and needs to be integrated …

The graduation curve is:

$$e_u = -Blv = -Bl\dot{x}$$

These are all *contactless* sensors !

Type (b) transducers are *reversible* and can become (with an appropriate design) ***loudspeakers*** !

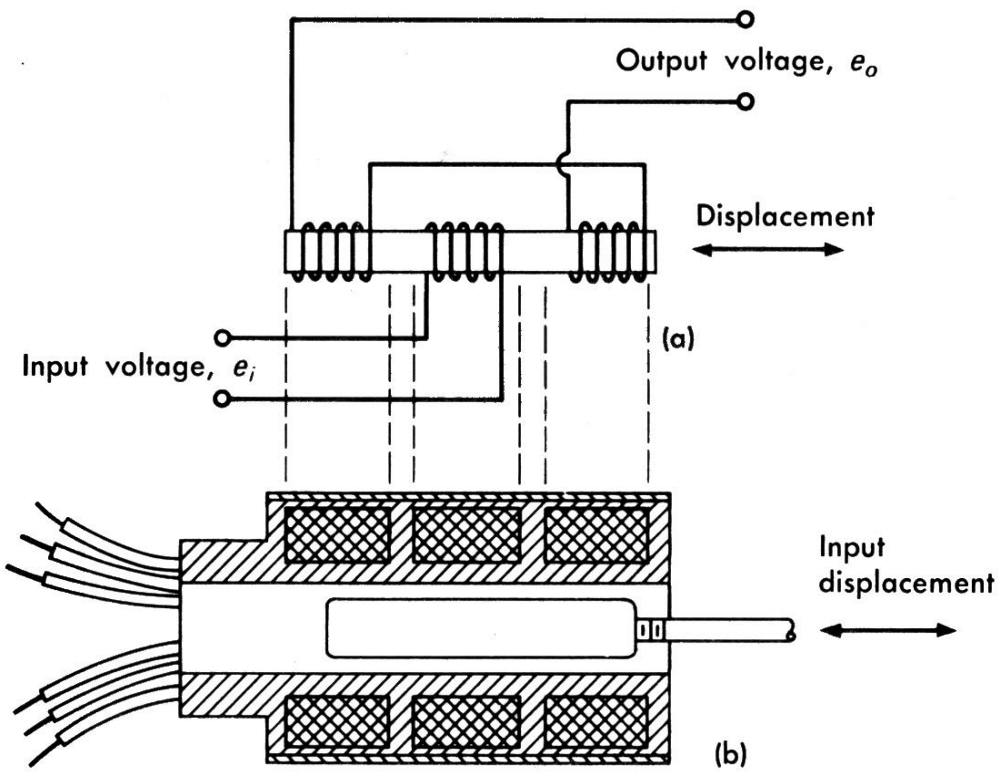
A typical application of this kind of transducers is the ***microphone pick-up*** …

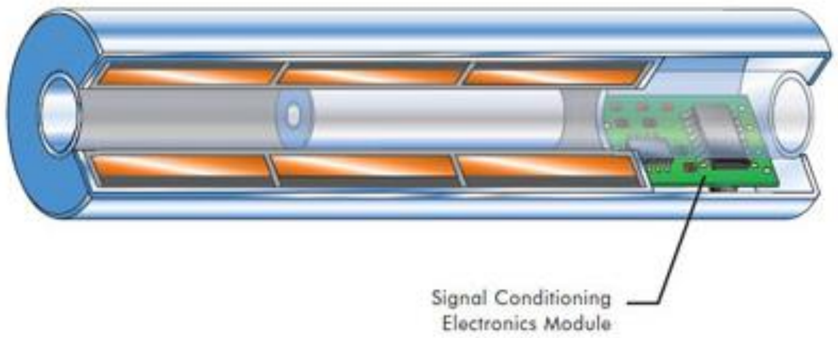
Proximity sensor comparison

Technology	Sensing range	Applications	Target materials
Inductive 	<4-40 mm	Any close-range detection of ferrous material	Iron Steel Aluminum Copper etc. 
Capacitative 	<3-60 mm	Close-range detection of non-ferrous material	Liquids Wood Granulates Plastic Glass etc. 
Photoelectric 	<1mm- 60 mm	Long-range, smalll or large target detection	Silicon Plastic Paper Metal etc. 
Ultrasonic 	<30 mm- 3 mm	Long-range detection of targets with difficult surface properites. Color/reflectivity insensitive.	Cellophane Foam Glass Liquid Powder etc. 

courtesy of "machinedesign.com"

The most important inductive transducer is the ***Linear Variable Dispalcement Transducer (LVDT)***





One primary and *two secondary coils* are wrapped as in figure (a). A high permeability (μ_r>10000) *inner nucleus* (b) picks up the displacement and engages the magnetic field produced by the primary coil …

Input voltage E_i at the primary coil is, of course, a.c.

Because the secondary coils are *winded series-opposing* the resulting output voltage E_0 will be differential:

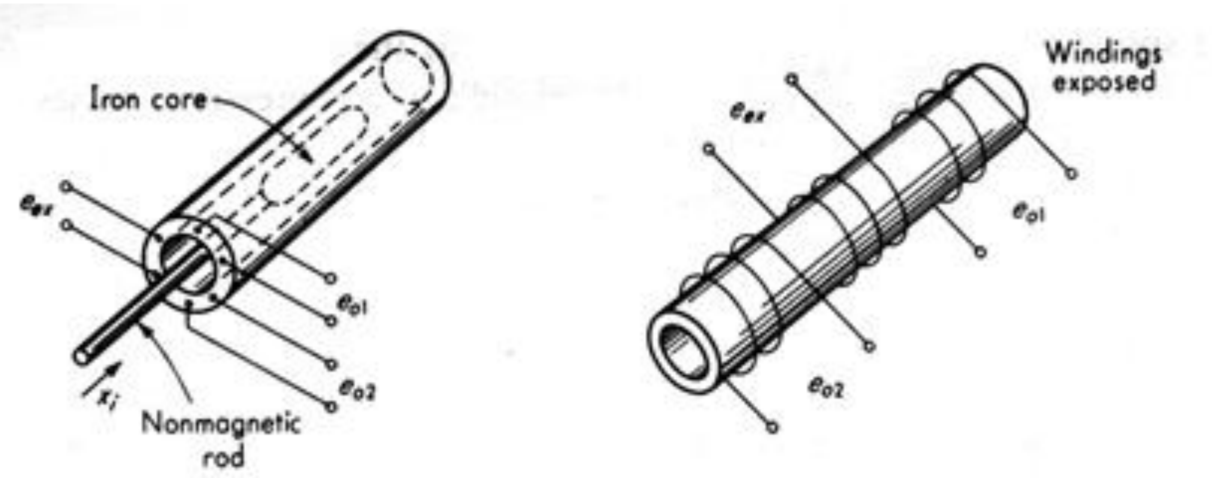
$$E_o = E_1 - E_2 = \left(M_{1p} - M_{2p}\right) \cdot \frac{di_p}{dt}$$

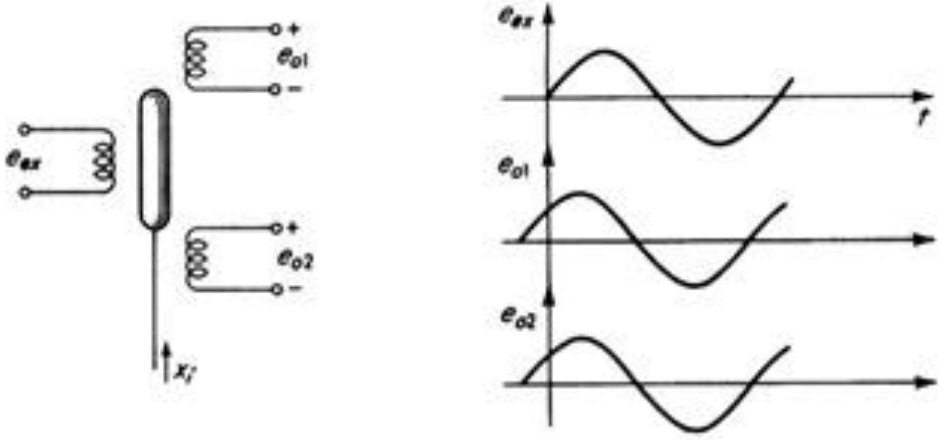
with the mutual inductance coefficients $M_{ip} = \sqrt{L_i L_p}$

and $L_i = \mu_o \mu_r \frac{N^2 S}{l}$ with i = 1 , 2

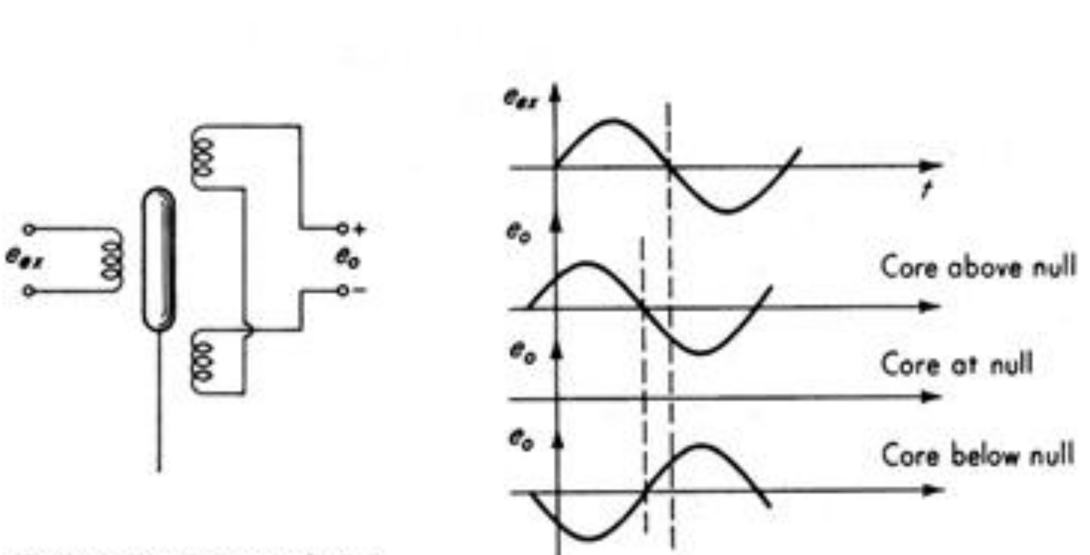
Note that the relationship above is valid for "every time instant" and is also preliminary form of the *LVDT graduation curve* !

The right figure explains what is happening when the ferromagnetic nucleus moves from the *central null position,* <u>copying</u> the external displacement. It can be easily seen that the LVDT is an electric “zero method” transducer !

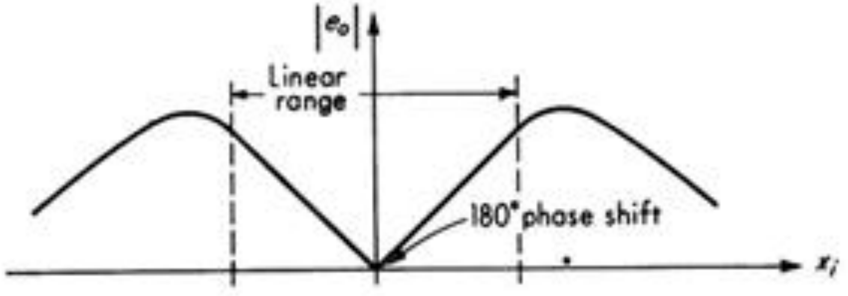




Core in null position

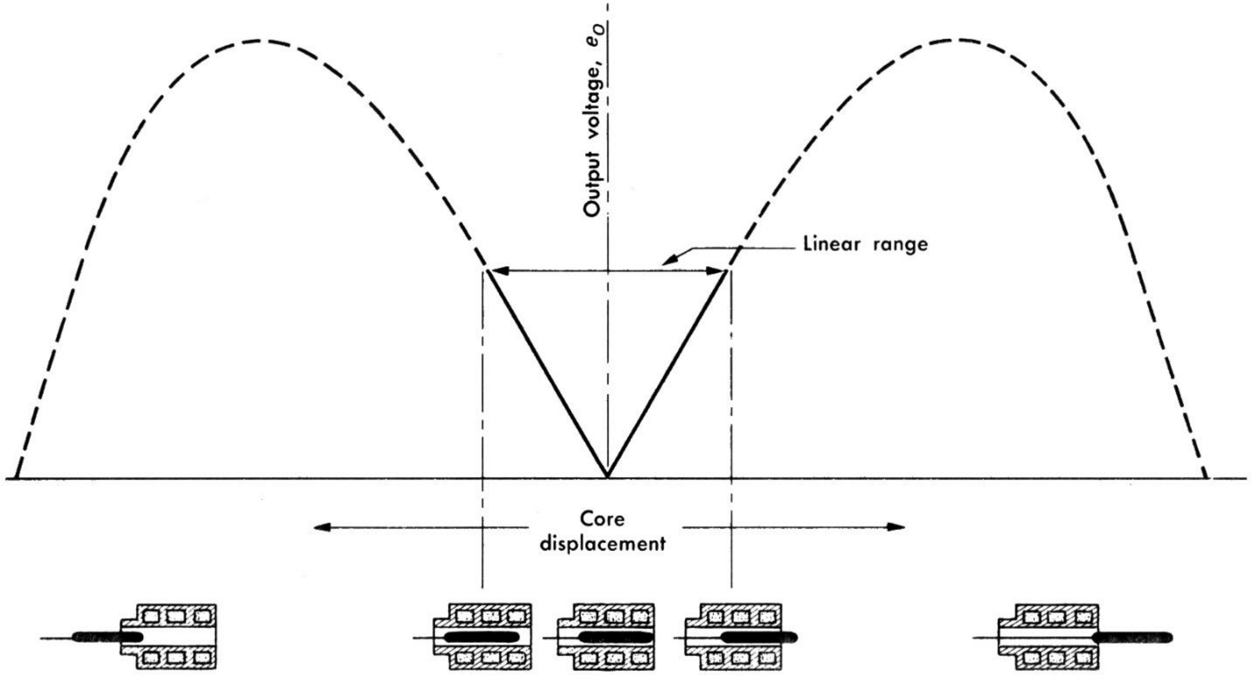


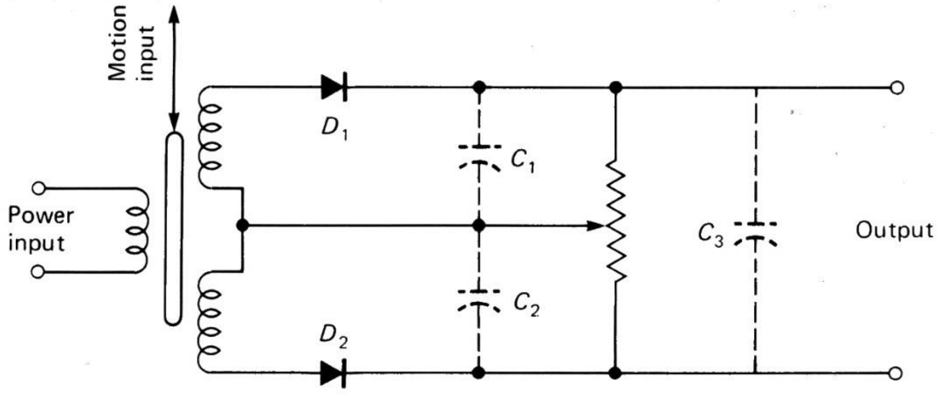
Series-opposing secondaries



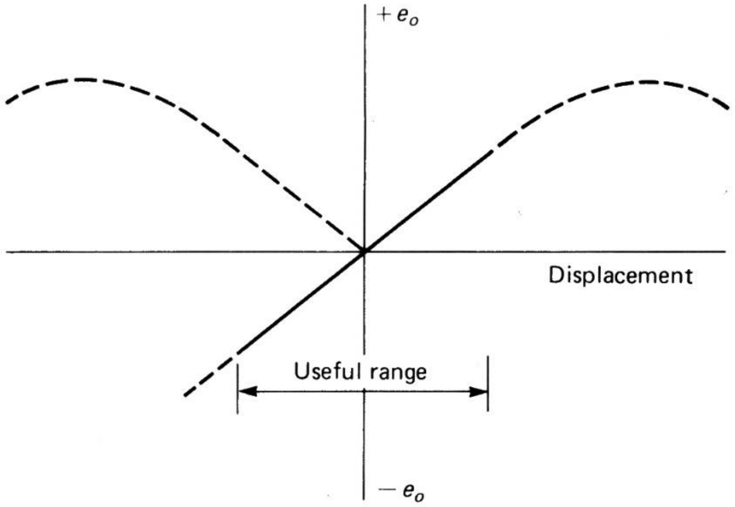
Differential transformer.

LVDT are **not** linear transducers but they can be considered liner in a limited range, near the central null position !



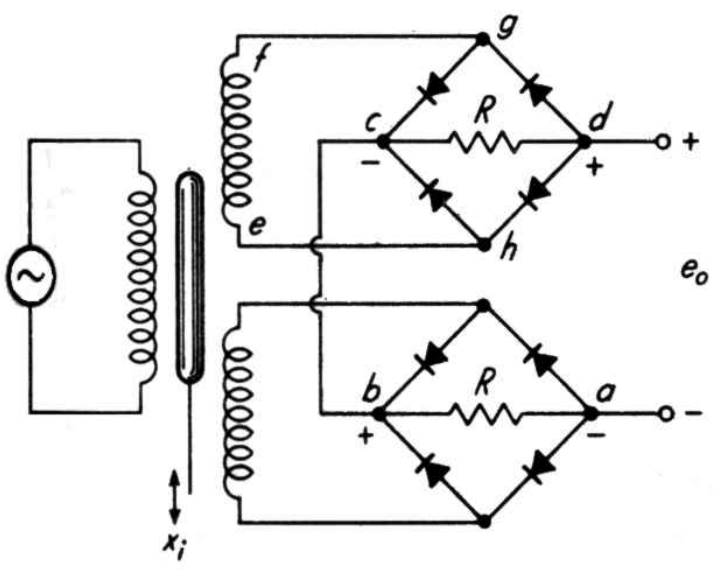


(a)



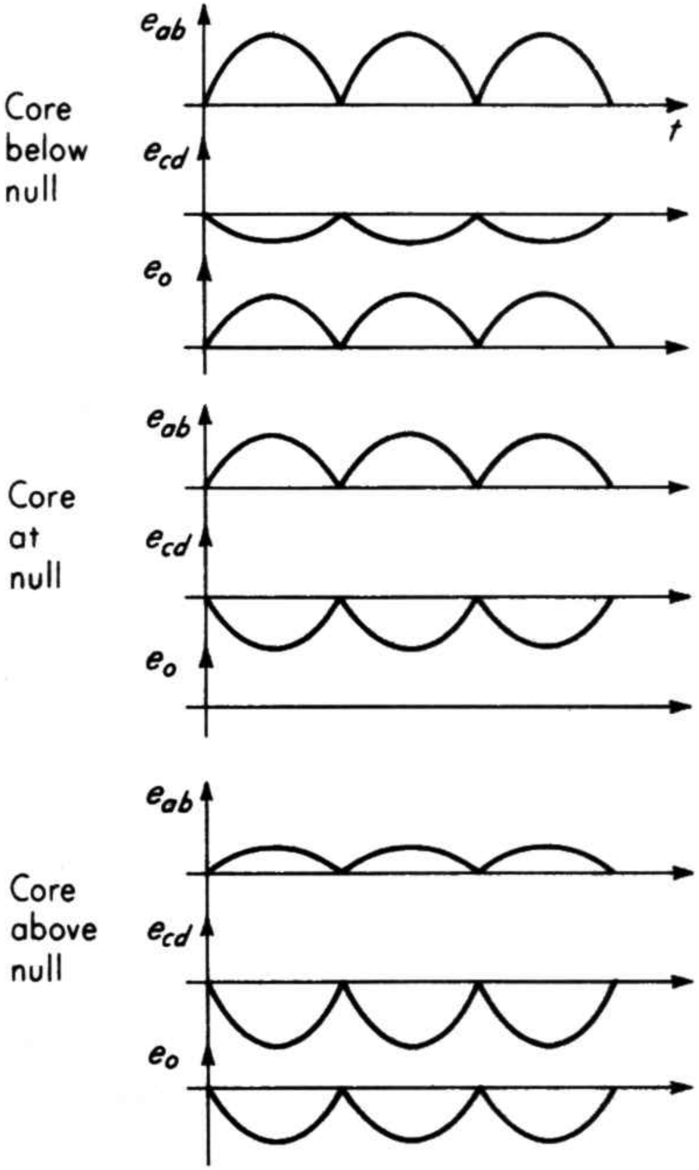
(b)

To read the output voltage, LVDT need a signal manipulation circuit that provides for the *rectification of the signal E_0* and a *phase discriminator* to understand if the displacement is happening towards the left or the right side !



$e_o = e_{ab} + e_{cd}$

$R \approx$ 1,000 to 10,000 Ω



Example of rectifier and phase demodulator …

Dynamic example:

LVDT with a 10g nucleus, measuring a displacement of ± 1mm at 1 kHz

Displacement: $x(t) = X_0 sen\omega t$

Acceleration (of the nucleus):

$\ddot{x}(t) = -\omega^2 X_0 sen\omega t = \omega^2 X_0 sen(\omega t - \pi)$

Whatever is the displacement we are measuring, to correctly pick up the ± 1mm amplitude at 1kHz frequency, the external displacement must apply to the nucleus in every time instant a force

$F(t) = m\ddot{x}(t) = m \cdot \omega^2 X_0 sen(\omega t - \pi)$

which is maximum at the inversion points: $F_{\max} = m\ddot{X}_0 = m \cdot \omega^2 X_0$

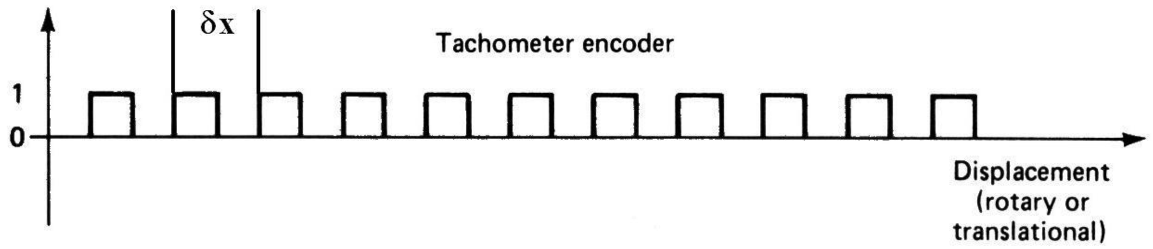
which actually is …

with $\omega = 2\pi f = 6,28 \times 10^3 s^{-1}$ and $X_0 = 10^{-3} m$

$$F_{\max} = 10^{-2} kg \times 39,4 \cdot 10^6 s^{-2} \times 10^{-3} m = 394 kgms^{-2} \cong 400N$$

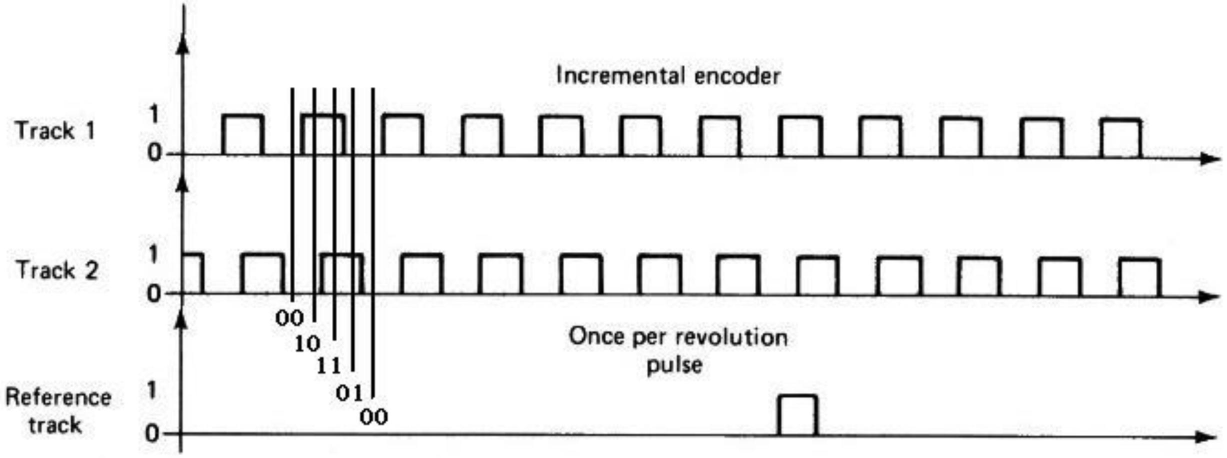
the ***dynamic loading effect*** …

Digital ENCODERS



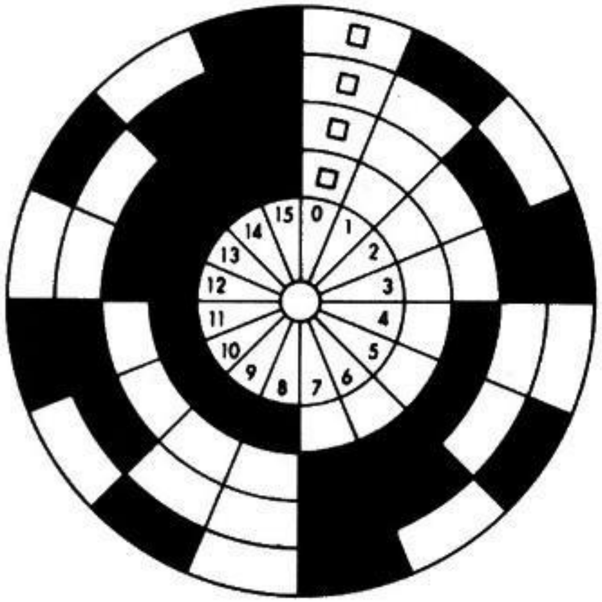
Encoders are “native digital” *motion transducers*. They are simply made by coupling a *notch trace* and a *notch reader* in relative motion by each other. The distance *δx* (pitch) between two consecutive notches is the fundamental design data of the encoder.

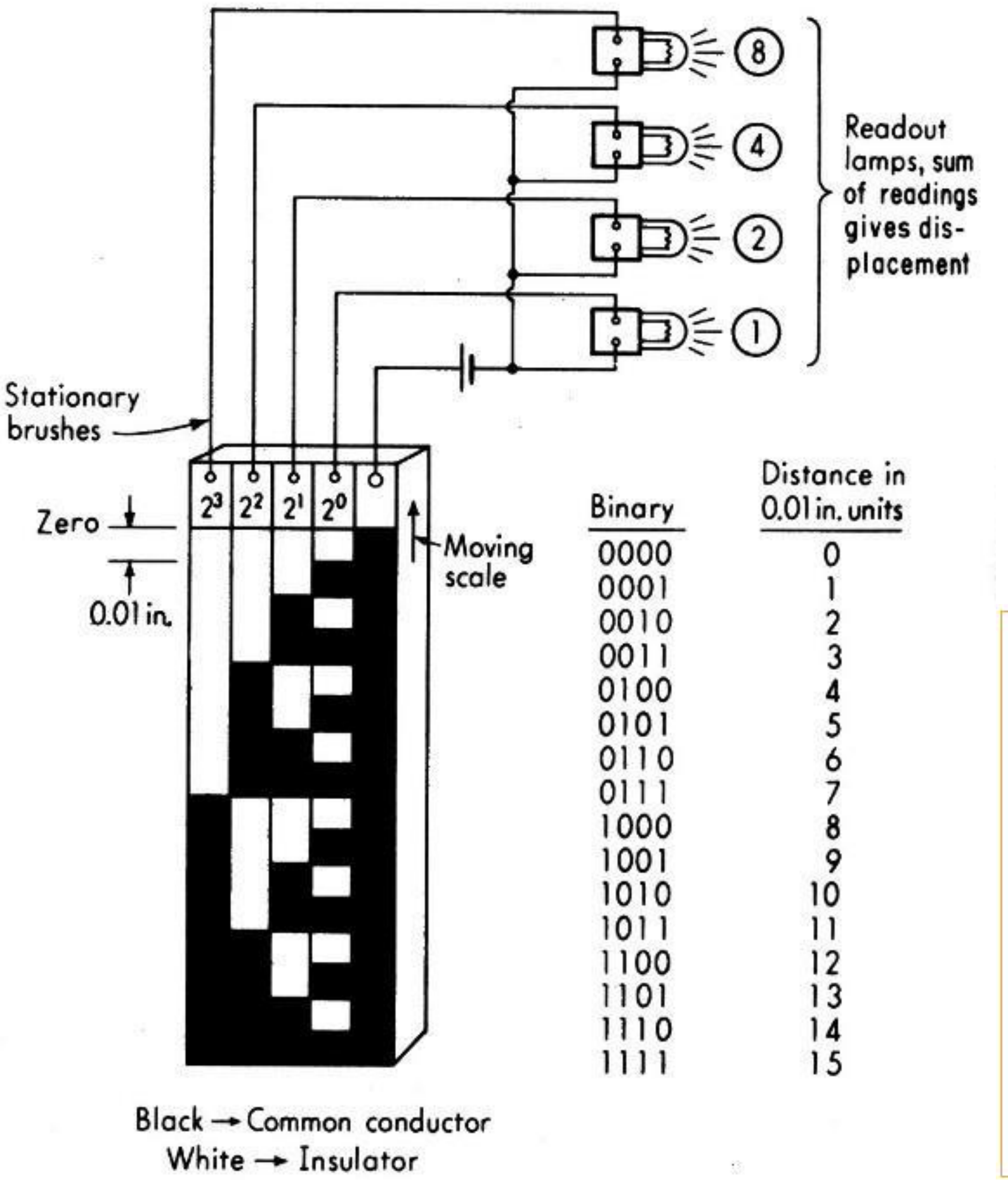
In ***tachometric encoders*** we just read the number of notches the sensor has counted and we have the distance: $x = n \times \delta x$. The *number of notches counted in each time unit* gives us also the velocity of the motion !
Spatial resolution is equal to $\delta x/2$ but NO information about motion verse is provided by such transducer !



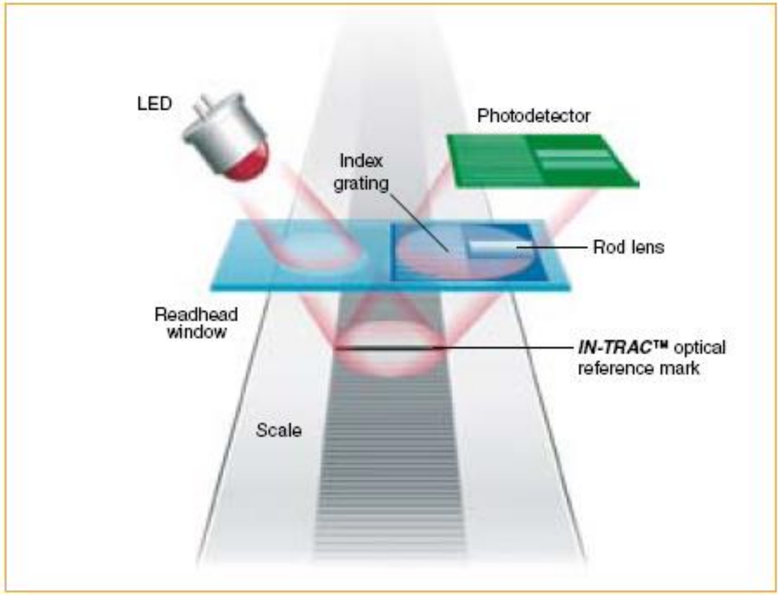
Incremental encoders can overcome this limit by using two notch traces, staggered each other with a pitch of *δx/4*.
This is also the new *enhanced resolution* of the transducer.
NO information about the absolute position is provided by such transducer !

Absolute encoders make a direct association between the *actual position* and a *binary code* !
They have as many notch traces (and readers) as the bit number that is needed !

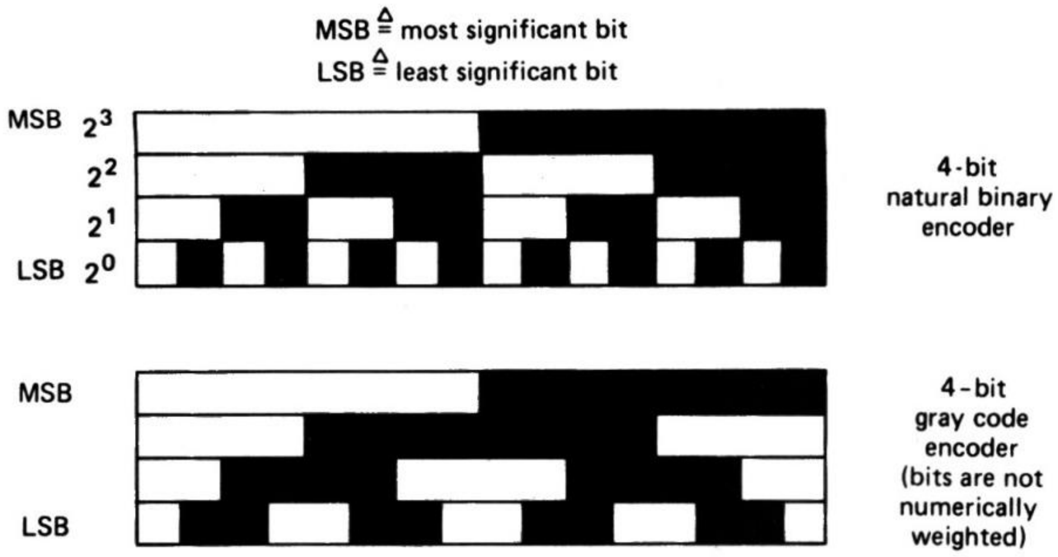


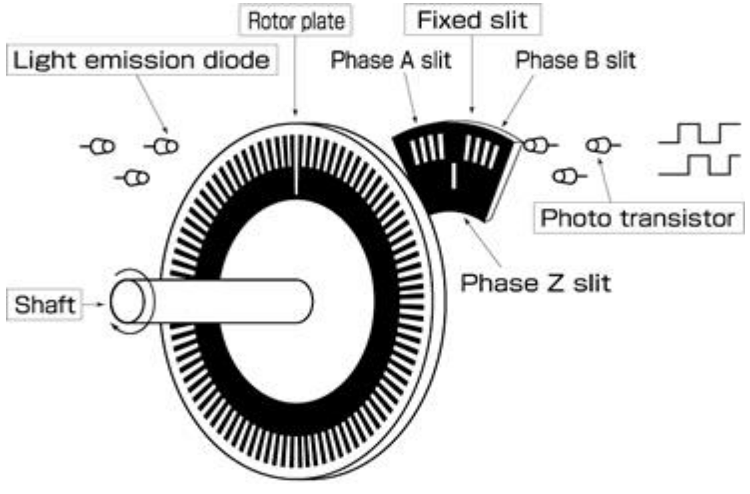


Translational and rotary encoders.

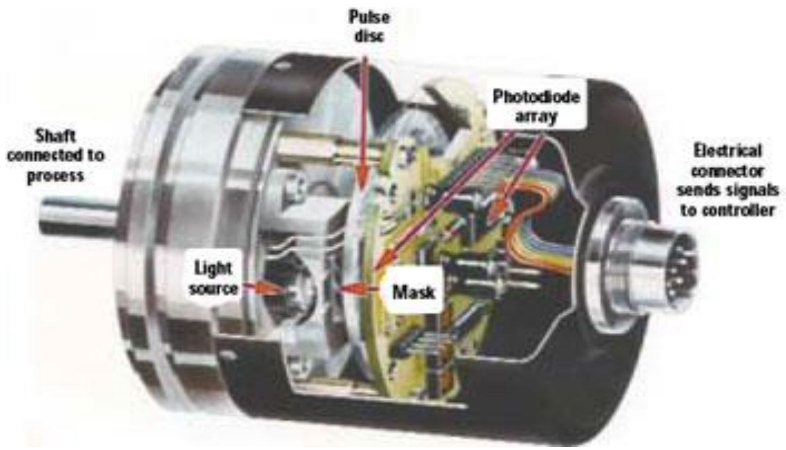


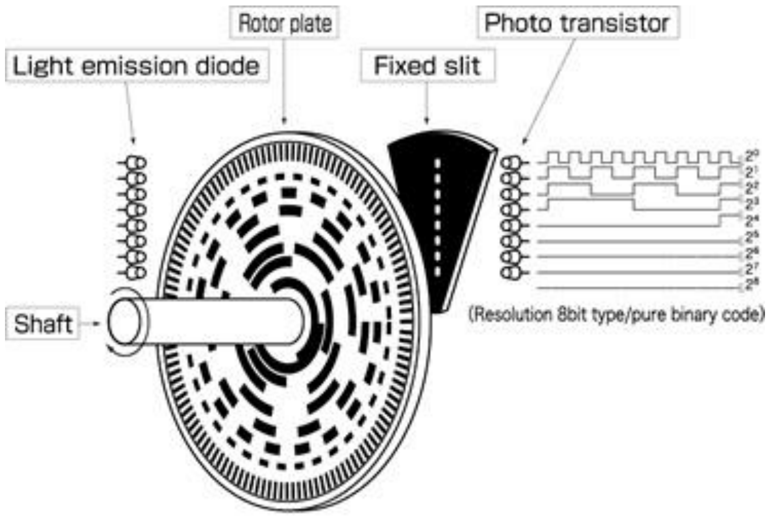
The ***natural binary code*** is not ideal for *encoders* because there are several transitions for which more that one bit is changing its state simultaneously ! This is not guaranteed in real devices and it can cause noise …
Therefore, encoders employ the ***grey binary code*** where, for every step, there is always only one bit at a time that changes its state !





Incremental Encoder Simplified Structure





Absolute Encoder Simplified Structure